

Microscopic quantum structure of black hole and vacuum versus quantum statistical origin of gravity

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Abstract

The Planckon densely piled model of vacuum is proposed. Based on it, the microscopic quantum structure of Schwarzschild black hole and quantum statistical origin of its gravity are studied. It is shown that thermodynamic temperature equilibrium and mechanical acceleration balance make the space-time of the black hole horizon singular and Casimir effect works inside the horizon. This effect makes the inside vacuum have less zero fluctuation energy than the outside vacuum, and a temperature difference as well as gravity as thermal pressure are created. A dual relation between inside and outside regions of the black hole is found. By the dual relation, an attractor behaviour of the horizon surface is unveiled. Outside horizon, there exist thermodynamic non-equilibrium and mechanical non-balance which lead to outward centrifugal energy flow and inward gravitation energy flow, their compensation establishes local equilibrium. The lost vacuum energy in negative gravitation potential regions has been removed to the black hole surface to form a spherical Planckon shell with the thickness of Planckon diameter. All the particles absorbed by the black hole have fallen down to the horizon and converted into spin 1/2 radiation quanta made of standing waves on the horizon sphere with the mean energy related to Hawking-Unruh temperature, thermodynamic equilibrium and mechanical balance keep them stable and be tightly bound in the horizon. The gravitation mass $2M$ and physical mass M of the black hole are calculated. The entropy of the black hole, calculated from the microscopic state number of the many-body system of radiation fermion quanta, is well consistent with Hawking. A radical modification of the temperature law of the black hole is made. The accelerating expansion of the universe yields the expansion cosmon and its energy density agrees with dark energy density.

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I. STRUCTURE OF VACUUM: PLANKON DENSELY PILED VACUUM MODEL

Black holes are typical objects in astrophysics, which show full nature of gravity[1–5]. In this article, we shall try to explore microscopic quantum structure of vacuum and quantum statistical origin of gravity by virtue of investigation of black holes.

During the search for microscopic foundation of the temperature as well as the four thermodynamic laws of black holes, it is naturally to conjecture that vacuum has a microscopic quantum structure and the thermodynamic laws of black holes are rooted in microscopic

quantum statistical physics.

To make the above conjecture concrete, we have proposed a novel model of vacuum which is piled up densely by extremely tiny Planck radiation quantum spheres and is a kind of a liquid crystal under mean field and semi-classical approximation. The Planck radiation quantum sphere called Planckon, is made of localized standing radiation waves on average with radius $r_p \approx 10^{-33}cm$, mass $m_p \approx 10^{-5}g$, energy $e_p \approx 10^{19}GeV$, and spin $s_p = \hbar/2$.

There are three ways to find the parameters of the Planckon. The first way requires that the Planckon is a black hole with its radius $r = 2Gm/c^2$, which is made of a quantum standing spherical wave with wave length $\lambda = 4\pi r$, wave vector $k = 2\pi/\lambda = 1/2r$, quantized energy $e(r) = \hbar kc = \hbar c/2r = \hbar\omega$, and mass $m(r) = e(r)/c^2 = \hbar/2rc$. From above requirements, one obtains radius, energy, mass, and spin of Planckon as follows:

$$r_p = \sqrt{\frac{G\hbar}{c^3}}, \quad e_p = \frac{\hbar c}{2r_p}, \quad m_p = \frac{1}{2}\sqrt{\frac{c\hbar}{G}}, \quad s_p = m_p c r_p = \frac{1}{2}\hbar \quad (1)$$

The second way requires that the Planckon has spin $\hbar/2$ and the balance between gravitation force and centrifugal force is established. The third one requires that the Planckon is a quantum standing spherical wave and the gravitation force and centrifugal force are balanced. All the three approaches reach the same results. Thus Planckon is the smallest microscopic quantum black hole with spin $\hbar/2$ and made of localized radiation standing wave. It is also the smallest microscopic quantum particle with the heaviest mass.

Summary of Planckon's parameters: scales of space-time $r_p = (\hbar G/c^3)^{1/2}$, $t_p = (\hbar G/c^5)^{1/2}$; volume $v_p = 4\pi r_p^3/3$; spin $s_p = \hbar/2$; mass $m_p = \frac{1}{2}(\hbar c/G)^{1/2} = \hbar/2cr_p$, energy $e_p = m_p c^2 = \hbar c/2r_p = \hbar\omega_p$, wave nature $\omega_p = c/2r_p$, $k_p = e_p/\hbar c = 1/2r_p$, $\lambda_p = 4\pi r_p$; zero energy: $e_{p0} = e_p/2 = \hbar c/4r_p$; zero energy density of Planckon: $\rho_{p0} = e_{p0}/v_p = \frac{3}{16\pi} \frac{c^4}{G^2 \hbar} = \frac{\rho_p}{2}$ ($e_{p0} = \rho_{p0} v_p = \frac{\hbar c}{4r_p}$); vacuum zero energy density is twice of that of Planckon (since vacuum is made of densely piled Planckons and each Planckon has two spin states): $\rho_v = \rho_p = 2\rho_{p0} = \frac{3}{8\pi} \frac{c^4}{G^2 \hbar}$, $\rho_v v_p = e_p = \frac{\hbar c}{2r_p}$; others: $Gm_p = r_p c^2/2$, $2Gm_p/c^2 = r_p$, $G = 6.6 \times 10^{-8} cgs$.

Microscopic structure of Planckon vacuum: there are two ways (phases) to densely pile vacuum by spherical Planckons[6]:

Face-center cubic crystal: pile layer order is ABCABCABC... pile period is ABC

Hexagon dense crystal: pile layer order is ABABABABAB... pile period is AB

For infinite crystal, the two phases are degenerate in energy, since a rotation of the C

layer of the face-center cubic by 60° reaches the Hexagon dense phase. However, for finite crystal, due to dislocation or defects, their energies become non-degenerate. For a cubic with side length R (for a sphere with radius R , the result is the same), the volume is $V = R^3$, the smallest dislocation volume is $\Delta V = 3R^2\Delta R = 6R^2r_p$ with smallest $\Delta R = 2r_p$. Hence $\frac{\Delta V}{V} \sim \frac{6r_p}{R}$. For mass density $\rho = \frac{m}{v}$, $\frac{\Delta\rho}{\rho} = \frac{6r_p}{R}$; for vacuum energy density $\rho = \rho_p$, dislocation energy is: $\Delta\rho v_p = Mc^2$, $e_p = \rho_p v_p = m_p c^2$, $\frac{M}{m_p} \sim \frac{6r_p}{R}$; for proton ($R_p \approx 10^{-13} \text{cm}$), dislocation mass is $M_p \sim \frac{6r_p}{R_p} m_p \approx 10^{-24}g$.

Vacuum elasticity coefficient: $K \approx \frac{e_p}{r_p^2} \approx \frac{10^{19} \text{GeV}}{10^{-66} \text{cm}^2} \sim 10^{82} \text{erg/cm}^2$

Vacuum transverse wave velocity c_ν equal to c : $c_\nu^2 = \frac{\mu_\nu}{\rho_\nu/c^2} = \frac{\mu_\nu c^2}{\rho_\nu} = c^2$ (where ρ_ν/c^2 is vacuum mass density), indicating that vacuum transverse stress μ_ν is equal to vacuum energy density ρ_ν : $\mu_\nu = \rho_\nu$.

Vacuum longitudinal stress K_ν : from $\rho_\nu \sim C/r_p^4 \propto C/v_p^{4/3}$, one gets $K_\nu = -v_p \frac{\partial(\rho_\nu c^2)}{\partial v_p} = \frac{4}{3}\rho_\nu = \frac{4}{3}\mu_\nu$

The solid property of vacuum comes from its extremely large mass density and stress coefficients, and its liquid property is due to the variability of localized radiation waves of Planckons. Thus the Planckon densely piled vacuum is a kind of liquid crystal.

II. MICROSCOPIC QUANTUM STRUCTURE OF SCHWARZSCHILD BLACK HOLE AND QUANTUM STATISTICAL ORIGIN OF TEMPERATURE AND GRAVITY

A. Quantum statistical meaning of the temperature at black hole horizon

Basic parameters:

Black hole radius

$$r_h = \frac{2GM}{c^2}, \quad g_{00}(r) = \left(1 - \frac{r_h}{r}\right) \quad (2)$$

Hawking-Unruh temperature[4, 5]

$$T_h = \frac{\hbar\kappa}{2\pi k_B c} \quad (3)$$

Surface gravity acceleration:

$$\kappa = \lim_{r \rightarrow r_h} \sqrt{\frac{g^{rr}}{g_{00}}} \frac{dg_{00}(r)}{dr} = \frac{c^2}{r_h} \quad (4)$$

To explore the quantum statistical meaning of the black hole and its gravity, we introduce the spherical standing radiation wave as vacuum excitation quantum as follows: wave length $\lambda_h = 4\pi r_h$ and wave vector $k_h = \frac{2\pi}{\lambda_h} = \frac{1}{2r_h}$, wave frequency $\omega_h = \frac{c}{2r_h} = 2\pi \frac{c}{\lambda_h} = 2\pi\nu_h$, quantum energy $e_h = \frac{\hbar c}{2r_h} = \hbar\omega_h$, quantum mass $m_h = e_h/c^2 = \hbar/2cr_h$, and spin $s_h = m_h cr_h = \frac{\hbar}{2}$. The radiation quantum moving in spherical horizon is a spin 1/2 fermion which is an astronomic dual of Planckon as follows: $r_p \leftrightarrow r_h$

Planckon quantum:

$$e_p = \frac{\hbar c}{2r_p}, \quad m_p = \frac{\hbar}{2cr_p}, \quad s_p = \frac{\hbar}{2}, \quad \lambda_p = 4\pi r_p \quad (5)$$

Radiation quantum:

$$e_h = \frac{\hbar c}{2r_h}, \quad m_h = \frac{\hbar}{2cr_h}, \quad s_h = \frac{\hbar}{2}, \quad \lambda_h = 4\pi r_h \quad (6)$$

According to the temperature Green's function for fermion[7, 8], the horizon temperature T_h and thermal energy $k_B T_h$ are related to the radiation quantum mean energy e_h

$$e_h/k_B T_h = \beta e_h = \pi \quad (\beta = 1/k_B T_h) \quad (7)$$

It turns to be

$$\pi k_B T_h = e_h = \frac{\hbar c}{2r_h} = \frac{\hbar \kappa}{2c}$$

or

$$T_h = \frac{\hbar \kappa}{2\pi c k_B}, \quad \kappa = \frac{2GM}{r_h^2} = \frac{c^2}{r_h} \quad (8)$$

which is exactly the Hawking-Unruh formulae.

This is the first step to explore the microscopic contents of Hawking-Unruh formulae and find the elementary quantum constituent of the quantum statistical system of gravity—the radiation quantum and its motion mode, energy, wave vector, and spin, as well as the relation between the statistical temperature T_h and the constituent mean energy e_h . The information is enough for a quantum statistical (ideal gas) system if the number of the constituents is provided. In the following, we shall explore that the black hole temperature and the radiation quantum energy can be found from the Casimir effect of the isolation effect of the black hole horizon.

It should be noted that in black hole horizon and for particles with speed of light, there exists a mechanical balance such that gravity acceleration is equal to centrifugal acceleration:

$$\kappa = a_{gr} = \frac{2GM}{r_h^2} = \frac{c^2}{r_h} = a_{centrif} \quad (9)$$

This observation leads to another way to find the black hole radius. If it is required that the particles moving in black hole horizon must be the radiation particles with speed of light and their gravity acceleration must balance the centrifugal acceleration so that the particles are confined in the horizon, then $a_{gr} = \kappa = \frac{2GM}{r_h^2} = a_{centrif} = \frac{c^2}{r_h}$, which leads to the expression of black hole radius $r_h = 2GM/c^2$. Since every particle in horizon is moving with speed of light, the space-time metrics become singular in horizon, $g_{00}(r_h) = 0$ and $g_{rr}(r_h) = \infty$. For spherical black hole the possible solutions are $g_{00}(r_h) = 1 - \frac{r_h}{r} = 1/g_{rr}(r) = 1 - \frac{2GM}{rc^2}$.

B. Microscopic structure of black hole and Casimir effect of spherical horizon isolation

In this subsection, we shall show that the black hole temperature and the radiation quantum energy can be found from the Casimir effect of the isolation effect of the horizon.

Since particles can not escape from horizon, the spherical surface of the black hole horizon becomes a cutoff surface for radial waves from inside of the sphere. From the wave equation with spherical symmetry inside black hole, the spin 1/2 radial excitation wave function $u(r)$ of the Planckon vacuum medium has the following boundary condition at the spherical surface of the black hole

$$\begin{aligned} u(r) &\xrightarrow{r \rightarrow r_h} \sin(2kr)/2kr \\ \sin(2kr_h) &= 0, \quad 2kr_h = n\pi, \quad k_n = \frac{n\pi}{2r_h} = nk_h \\ k_h &= \frac{2\pi}{\lambda_h} = \frac{\pi}{2r_h}, \quad \lambda_h = 4r_h \end{aligned} \quad (10)$$

This leads to a discrete energy spectrum for the inside radial modes which decrease the vacuum zero energy density inside.

Zero energy density of vacuum outside. Outside the horizon, the zero energy density of vacuum contributed from one kind of states of the fermion-like modes can be calculated as the fermion gas model as follows

$$(2\pi)^{-2} \hbar c V \int_0^{\pi k_p} k^3 dk = \frac{\hbar c V}{4(2\pi)^2} (\pi k_p)^4 \quad (11)$$

where πk_p is the fermion wave vector related to Planck wave vector. The zero energy density of vacuum outside is

$$\rho_{p0} = \frac{\hbar c}{4(2\pi)^2} (\pi k_p)^4 \quad (12)$$

$$k_p = \frac{1}{2r_p} = \frac{N_p}{2r_h}, \quad \lambda_p = 4\pi r_p \quad (13)$$

$$N_p = \frac{r_h}{r_p} = \frac{10^5 cm}{10^{-33} cm} \sim 10^{38} \quad (14)$$

Zero Energy density of vacuum inside. Since the spectrum is discrete, the integration should be replaced by summation and the fermion wave vector is $k_N = N\pi/2r_h = N\pi k_p = N\pi/2r_p$, $N = r_h/r_p = N_p$. The results are

$$\sum_{n=0}^{N-1} n^3 = \frac{1}{4} N(N-1)^2 \approx \frac{N^4}{4} (1 - \frac{2}{N}) = \frac{N_p^4}{4} (1 - \frac{2}{N_p}) \quad (15)$$

$$(2\pi)^{-2} \hbar c V (\frac{\pi}{2r_h})^4 \sum_{n=0}^{N-1} n^3 = \frac{\hbar c V}{4(2\pi)^2} (\frac{\pi}{2r_h})^4 (N(N-1))^2 \approx \frac{cV}{4(2\pi)^2} (\pi k_p)^4 (1 - \frac{2}{N_p}) \quad (16)$$

The zero energy density of vacuum inside is

$$\rho_{h0} = \frac{\hbar c}{4(2\pi)^2} (\pi k_p)^4 (1 - \frac{2}{N_p}) \quad (17)$$

The difference of zero energy densities of vacuum between outside and inside regions is

$$\Delta\rho_{p0} = \rho_{p0} - \rho_{h0} = \frac{\hbar c}{4(2\pi)^2} (\pi k_p)^4 [1 - (1 - \frac{2}{N_p})] = \frac{2\hbar c}{4(2\pi)^2} (\pi k_p)^4 / N_p = 2\rho_{p0}/N_p, \quad (18)$$

$$\Delta\rho_{p0} = \frac{2}{N_p} \rho_{p0}, \quad \rho_{p0} = \frac{\hbar c}{4(2\pi)^2} (\pi k_p)^4 \quad (19)$$

$$\Delta\rho_{p0}/\rho_{p0} = \frac{2}{N_p} = \frac{2r_p}{r_h} = \frac{2m_p}{m_h} \sim \frac{10^{-33}}{10^5} = 10^{-38} \quad (20)$$

$\Delta\rho_{p0}$ is the zero energy density loss of the inside vacuum due to the cutoff boundary condition at the spherical horizon-the **Casimir effect**. [7, 8]

Since vacuum consists of densely piled Planckons and vacuum energy density is the Planckon energy density $\rho_v = \rho_p$ and $e_{p0} = \rho_{p0} v_p$. Inside black hole, each Planckon of one kind of spin states loses zero energy

$$\Delta e_{p0} = \Delta\rho_{p0} v_p = \frac{2e_{p0}}{N_p} = \frac{c\hbar}{2r_h} = e_h, \quad (21)$$

$$e_{p0} = \frac{\hbar c}{4r_p}, \quad e_p = \frac{\hbar c}{2r_p}, \quad N_p = \frac{r_h}{r_p}$$

which is exactly the mean energy e_h of the radiation quantum of the spherical standing wave of the black hole. This in turn implies a temperature decrease ΔT . According to the temperature Green's function for fermion[7, 8], the mean energy e_h and the temperature decrease ΔT has the relation

$$\Delta e_{p0}/k_B \Delta T = \beta e_h = \pi, \quad \pi k_B \Delta T = e_h = \frac{c\hbar}{2r_h} \quad (22)$$

$$\Delta T = \frac{\hbar c}{2\pi c k_B r_h} = \frac{\hbar \kappa}{2\pi c k_B} \quad (23)$$

where $\kappa = 2GM/r_h^2 = c^2/r_h$. This is exactly the Hawking-Unruh Formulae for the black hole, which is obtained from a different principle- the Casimir effect due to the geometric boundary condition for inside vacuum excitation waves. Huge number of radiation quanta with mean energy e_h and moving in spherical horizon will produce outward centrifugal energy flow and corresponding centrifugal temperature $T_h = \frac{\hbar \kappa}{2\pi c k_B} (\kappa = \frac{c^2}{r_h})$ which balances the inward temperature decrease ΔT due to Casimir effect, namely $\Delta T = T_h$. Thus a thermodynamic equilibrium is established in horizon.

C. Black hole mass: gravitation mass and physical mass

The black hole absorbs particles from outside. The following calculations show that the absorbed particles are changed into spin 1/2 radiation quanta stored in horizon and with the mean energy $e_h = \hbar c/2r_h$ and the mean zero energy $e_{h0} = \hbar c/4r_h$. The number of radiation quanta is exact the number of Planckons in the horizon which consists of only one Planckon layer with the thickness of Planckon diameter $d = 2r_p$. This means that each Planckon in horizon has a resident radiation quantum and the host Planckon provides its position quantum number to the resident radiation quantum as a hidden quantum number. This position hidden quantum number resolves the Pauli exclusion problem for the super condensate in horizon of the extremely large number of the fermion-type radiation quanta with extremely long wave length $\lambda_h = 4\pi r_h$.

For the densely piled Planckon vacuum, The number N_h of Planckons in horizon layer is equal to the area of the sphere $4\pi r_h^2$ divided by the area of the largest circle of the Planckon

sphere πr_p^2 :

$$N_h = \frac{4\pi r_h^2}{\pi r_p^2} = 4\left(\frac{r_h}{r_p}\right)^2 \quad (24)$$

The total mass M_A of the zero energy mass of the radiation quanta in horizon is N_h times radiation quantum zero energy mass $m_{h0} = e_{h0}/c^2 = \frac{\hbar}{4cr_h}$,

$$M_A = m_{h0}4\left(\frac{r_h}{r_p}\right)^2 = \frac{\hbar}{cr_h}\left(\frac{r_h}{r_p}\right)^2 = \frac{\hbar r_h}{cr_p^2} = \frac{\hbar 2GM}{(G\hbar/c^3)c^3} = 2M \quad (25)$$

which is twice of the black hole mass M , half of which (M_A) is used to compensate the negative vacuum (negative gravity potential) energy mass.

To understand how the twice energy is formed, consider a particle from infinite with energy $\hbar\omega_0$ and reaching the black hole surface with energy $\hbar\omega(r_h)$ and mass $m(r_h) = \hbar\omega(r_h)/c^2$. Newtonian Potential energy is $-\frac{\hbar\omega(r_h)}{c^2} \times \frac{GM}{r_h}$ (r-dependence of mass $m(r)$ plays the role of general relativistic effect). According to energy conservation, one has $\hbar\omega(r_h) - \frac{\hbar\omega(r_h)}{c^2} \times \frac{GM}{r_h} = \hbar\omega_0$ and $\hbar\omega(r_h) = \hbar\omega_0/(1 - \frac{GM}{c^2 r_h}) = 2\hbar\omega_0$. So as the particle reaches the horizon, its energy and mass are doubled.

The same result is obtained for a particle at infinite with rest energy $m_0 c^2$ and at horizon with energy $m(r_h)c^2$: $m(r_h)c^2 - m(r_h)c^2 \times \frac{GM}{c^2 r_h} = m_0 c^2$ and $m(r_h)c^2 = m_0 c^2/(1 - \frac{GM}{c^2 r_h}) = 2m_0 c^2$. The energy and mass are also doubled as it reaches horizon.

Since half of the energy $M_A c^2$ is used to compensate the negative vacuum energy loss, the physical mass of the black hole is the residual mass M . However the gravitation mass $M_A = 2M$ manifests itself as general relativistic effect in Einstein potential $\phi(r) = -\frac{2GM}{r}$ and gravity strength $\kappa(r_h) = \frac{2GM}{r_h^2}$.

In what follows, let us answer the question where the lost energy of vacuum inside black hole goes. As shown in sect. II-B, inside horizon each Planckon loses energy $\Delta e_{p0} = e_h = e_p(\frac{r_p}{r_h}) = 2e_{p0}(\frac{r_p}{r_h})$ and mass $\Delta m_{p0} = \Delta e_{p0}/c^2$. In Planckon densely piled vacuum, the number of Planckons inside horizon is $N_v = (\frac{4\pi r_h^3}{3})/(\frac{4\pi r_p^3}{3}) = (\frac{r_h}{r_p})^3$. For one kind of spin states, all the Planckons lose energy and mass totally amounting to $E_T = N_v \Delta m_{p0} c^2$ and $M_T = N_v \Delta m_{p0}$:

$$\begin{aligned} E_T &= \Delta e_{p0} \left(\frac{r_h}{r_p}\right)^3 = 2e_{p0} \left(\frac{r_h}{r_p}\right)^2 = \frac{1}{2} e_{p0} N_h, \\ M_T &= E_T/c^2 = 2m_{p0} \left(\frac{r_h}{r_p}\right)^2 = \frac{1}{2} m_{p0} N_h \end{aligned} \quad (26)$$

Since the Planckon inside horizon has two spin states and $N_h = 4r_h^2/r_p^2$ is the number of Planckons in the spherical surface layer, $2E_T$ and $2M_T$, contributed from all the Planckon losed energy and mass inside horison, are exact the energy and mass of Planckon layer at the spherical surface with one kind of spin states. It will be showed in sect.III that the vacuum energy and mass loss due to presence of gravitation outside horizon which is dual to the inside one, is also removed to the spherical surface to form another layer of Planckons with different spin state.

D. Microscopic origin of black hole entropy

It is showed that black hole mass M and energy Mc^2 come from radiation quanta in horizon, which are spin 1/2 fermions with the mean energy $e_h = \hbar c/2r_h$ and the mean zero energy $e_{h0} = \hbar c/4r_h$. The particles absorbed by the black hole have changed into radiation quanta and stored their information by virtue of microscopic states of radiation quanta. The number of radiation quanta in horizon Planckon layer is $N_h = 4r_h^2/r_p^2$. Each radiation quantum has two states. The total number of microscopic states of the many body system as an ideal gas with N_h fermions is $\Omega = 2^{N_h}$. According to the microscopic ensemble of quantum statistical physics, the entropy of black hole is:

$$S_h = k_B \ln \Omega = k_B \ln 2 \times N_h = k_B \ln 2 \times \frac{4r_h^2}{r_p^2} = \ln 2 \times \frac{k_B A_h c^3}{\pi \hbar G} = 0.22 \frac{k_B A_h c^3}{\hbar G} \quad (27)$$

where the black hole horizon area $A_h = 4\pi r_h^2$ and $r_p^2 = G\hbar/c^3$. The above result is comparable with Hawking, Rovelli, and Shao as follows: Hawking[4] $S_h = 0.25k_B A_h c^3/\hbar G$, Rovelli[8] $S_h = k_B A_h c^3/16\pi\hbar G$, and Shao[9] $S_h = \ln 2 \times k_B A_h c^3/8\pi\hbar G$.

E. Thermodynamic equilibrium and mechanical balance in horizon

For the ideal gas of radiation quanta with mean energy e_h in the horizon shell, there exist thermodynamic equilibrium and mechanical balance as follows. Inward Casimir-gravity temperature $\pi k_B \Delta T = \hbar \kappa/2c = \hbar c/2r_h (\kappa = 2GM/r_h^2 = c^2/r_h)$ and outward centrifugal temperature $\pi k_B T_{centrif} = e_h = \hbar c/2r_h = \hbar \kappa/2c (e_h = \hbar c/2r_h)$ are in thermodynamic equilibrium:

$$T_{gr} = T_{centrif} (T_{gr} = \Delta T) \quad (28)$$

Gravitation acceleration $a_{gr} = \kappa = 2GM/r_h^2 = c^2/r_h$ and centrifugal acceleration $a_{centrif} = c^2/r_h$ are in mechanical balance:

$$a_{gr} = a_{centrif}, \quad F_{gr} = ma_{gr} = F_{centrif} = ma_{centrif} \quad (29)$$

The above thermodynamic equilibrium and mechanical balance imply that only the radiation quanta with speed of light can exist and be tightly bound in the horizon, the non-radiation particles with speed less than c can not stay in horizon stably. The outward centrifugal energy flow is balanced by the inward gravity flow, so the radiation quanta in horizon are in thermodynamic equilibrium. The thermodynamic equilibrium and mechanical balance make the vacuum space-time of the horizon area singular, and the horizon surface becomes a cutoff boundary for the interior radial wave modes.

F. Gravitation effect of inside radiation quanta and attractor behaviour of horizon

We have shown that inside the black hole, each Planckon loses energy and creates a radiation quantum hole with mean energy e_h which corresponds to a constant inside temperature $\pi k_B \Delta T = e_h = \hbar c/2r_h = \hbar \kappa/2c$ and gravitation acceleration $\kappa = c^2/r_h$. From $\kappa = -\frac{d\phi(R)}{dR}$ one gets Einstein potential $\phi(R) = -c^2 R/r_h$ which is quite different from the conventional inside potential $\phi(R) = -c^2 r_h/R$ ($r < r_h$). From $\phi(R)$, we find that no singularity exists inside and its radial acceleration points to the horizon surface. It is well known that outside the horizon, the radial gravitation acceleration also points to the horizon surface. Thus the two-side gravity forces make the horizon surface become a special surface with attractor behavior. In Appendix A, it is shown that there is a dual relation between inside and outside regions. From the dual transformation $R = r_h^2/r$, one can obtain the outside Einstein potential $\phi(r) = -\frac{2GM}{r}$ from the inside one $\phi(R)$. The dual relation indicates that based on the physics of Casimir effect, the inside gravitation potential and gravitation acceleration as well as gravitation temperature have a completely different physical meaning in contrast to the conventional explanation.

III. MICROSCOPIC STRUCTURE OF EINSTEIN POTENTIAL OUTSIDE

In last subsection, we have obtained the outside Einstein potential $\phi(r)$ from the inside one which is obtained microscopically by using quantum statistical physics of ideal radiation quantum gas. In this section, we shall derive this potential by Laplace equation for local equilibrium temperature[11].

From the above discussion we know that the gravitation temperature from negative Einstein potential corresponds to the mean energy $e(r)$ of radiation quantum holes, which is related to potential energy $\Phi(r)$: $-\Phi(r) = e(r)$ and satisfies the period condition of temperature Green's function for fermion:

$$e(r)/k_B\Delta T(r) = \beta(r)e(r) = \pi \quad (30)$$

or

$$\Phi(r)/\pi = -k_B\Delta T(r) \quad (31)$$

The Laplace equation for local equilibrium temperature is [11]:

$$\nabla^2\Phi(r) = \pi k_B\nabla^2\Delta T(r) = 0 \quad (32)$$

The solution is

$$\Phi(r) = \frac{C}{r} \quad (33)$$

From boundary condition at r_h : $\Phi(r_h) = C/r_h$ and

$$\Phi(r_h)/\pi = -k_B\Delta T(r_h) = -\frac{\hbar\kappa}{2\pi c} = -\frac{\hbar c}{2\pi r_h} \quad (34)$$

$$\kappa = \frac{2GM}{r_h^2} = \frac{c^4}{2GM} = \frac{c^2}{r_h} \quad (35)$$

one obtains $\Phi(r_h) = -e(r_h) = -\hbar c/2r_h$, $C = -\hbar c/2$,and

$$\Phi(r) = -\frac{\hbar c}{2r}, \quad e(r) = \frac{\hbar c}{2r}, \quad k_B\Delta T(r) = k_B\Delta T(r_h)\frac{r_h}{r} \quad (36)$$

However $\Phi(r)$ is not Einstein potential $\phi(r) = -\frac{2GM}{r}$, instead it is the potential energy of a radiation quantum with mass $m_h = \frac{\hbar}{2cr_h}$ and energy $e_h = \frac{\hbar c}{2r_h}$: $\Phi(r) = -\frac{2GMm_h}{r} = -\frac{\hbar c}{2r} = -e(r)$. So the Einstein potential outside should be:

$$\phi(r) = \frac{\Phi(r)}{m_h} = -\frac{c^2 r_h}{r} = -\frac{2GM}{r} \quad (37)$$

We have shown that inside horizon, the Casimir effect leads to vacuum energy loss and creation of the spin 1/2 radiation quantum hole with mean energy $e_h = \frac{\hbar c}{2r_h}$ which is related to the temperature and gravity strength by the relation:

$$e_h/\pi = k_B T(r_h) = \frac{\hbar \kappa(r_h)}{2\pi c}, \quad (\kappa(r_h) = \frac{2GM}{r_h^2} = \frac{c^2}{r_h}) \quad (38)$$

Similarly, outside horizon, the negative gravitation potential $\phi(r)$ and the gravitation strength $\kappa(r_\kappa) = \frac{2GM}{r_\kappa^2} = \frac{c^2}{r_\kappa}$ also lead to vacuum energy loss and creation of the spin 1/2 radiation quantum hole with mean energy

$$e(r_\kappa) = \frac{\hbar c}{2r_\kappa}, \quad (r_\kappa = r \frac{r}{r_h}) \quad (39)$$

which is related to the temperature and gravity strength by the relation

$$e(r_\kappa)/\pi = k_B T(r_\kappa) = \frac{\hbar \kappa(r_\kappa)}{2\pi c}, \quad \kappa(r_\kappa) = \frac{c^2}{r_\kappa}, \quad T(r_\kappa) = \frac{\hbar c}{2\pi k_B r_\kappa} \quad (40)$$

This is the generalized Hawking-Unruh formulae from inside to outside regions.

Outside horizon, let us consider the radiation quantum as a standing wave moving on r -sphere. This radiation quantum will have mean energy $e(r) = \hbar c/2r$, its gravitation acceleration $\kappa(r_\kappa) = 2GM/r^2 = c^2/r_\kappa$ and gravitation temperature $T(r_\kappa) = \frac{\hbar c}{2\pi k_B r_\kappa}$. Its centrifugal acceleration is $a_{centrif} = \frac{c^2}{r}$ and centrifugal temperature is $T(r) = \frac{\hbar c}{2\pi k_B r}$. Since $\frac{c^2}{r_\kappa} < \frac{c^2}{r}$ and $T(r) > T(r_\kappa)$, the radiation quantum $e(r)$ can not stay in r -sphere stably, it will leave r -sphere. From the generalized relations Eq.(39 - 40) and the above consideration, we find that there exist thermodynamic non-equilibrium and mechanical non-balance on the r -sphere, namely the inward gravitation temperature $T(r_\kappa)$ is smaller than the outward centrifugal temperature $T(r)$, and the centrifugal acceleration $a_{centrif}(r) = c^2/r$ is larger than the gravitation acceleration $a_{gr}(r_\kappa) = \kappa(r_\kappa) = \frac{c^2}{r_\kappa}$. These thermodynamic non-equilibrium and mechanical non-balance lead to two kinds of radiation quantum energy flows with opposite directions: outward centrifugal energy flow and inward gravitation energy flow, their compensation establishes local equilibrium. These processes and flow lines should be studied by using the geodesic equations for zero geodesic trajectories of radiation quanta.

Now, let us answer the question where the lost energy of vacuum outside black hole goes. Outside horizon, the Einstein potential and its gravitation acceleration are:

$$\phi(r) = -\frac{2GM}{r} = -\frac{c^2 r_h}{r}, \quad \kappa(r_\kappa) = \frac{2GM}{r^2} = \frac{c^2 r_h}{r^2} = \frac{c^2}{r_\kappa}$$

$$r_\kappa(r) = r\left(\frac{r}{r_h}\right) > r \quad (41)$$

We have shown in Eq.(39 - 40) that outside horizon, the energy loss of vacuum creates the spin 1/2 radiation quantum hole with mean energy $e(r_\kappa) = \hbar c/2r_\kappa$ (its zero energy is $e_0(r_\kappa) = \hbar c/4r_\kappa$) and induces statistical temperature $T(r_\kappa)$. Since the radiation quanta in r -sphere shell and in horizon sphere shell are projectively connected, the number of radiation quanta in r -sphere shell is the same as in horizon sphere shell, namely the number of quanta in r -sphere shell is also $N_h = 4\left(\frac{r_h}{r_p}\right)^2$. The total zero energy of the radiation quanta in r -sphere shell with thickness $d = 2r_p$ is

$$dE(r) = e_0(r_\kappa)N_h \frac{dr}{2r_p} = \frac{\hbar c}{8r_\kappa} \times 4\left(\frac{r_h}{r_p}\right)^2 \frac{dr}{r_p} = \frac{1}{2}\hbar c\left(\frac{r_h}{r_p}\right)^3 \frac{dr}{r^2} \quad (42)$$

To obtain the total zero energy of all the radiation quanta with one kind of spin states outside the horizon, Eq.(42) should be integrated from r_h to ∞ ,

$$E_T = \int_{r_h}^{\infty} dE(r) = \frac{1}{2}\hbar c\left(\frac{r_h}{r_p}\right)^3 \int_{r_h}^{\infty} \frac{dr}{r^2} = \frac{\hbar c}{8r_p} 4\left(\frac{r_h}{r_p}\right)^2 = \frac{1}{2}e_{p0}N_h \quad (43)$$

Since each radiation quantum has two spin states, $2E_T$ is exactly the zero energy of the Planckons with one kind of spin states in the sphere layer at r_h . Thus outside the horizon, the vacuum energy loss manifesting itself as both negative gravitation potential and radiation quantum hole $e_0(r_\kappa)$, has been removed to spherical Planckon layer with one kind of spin, just like the inside vacuum energy loss manifesting itself as radiation quantum hole e_h has also been removed to the same layer with opposite spin state. In appendix A , by dual transformation, we do the same calculation and produce the same result.

IV. GRAVITATION POTENTIAL OF BLACK HOLE IN ACCELERATING UNIVERSE

For accelerating universe, the temperature depends on time in the way as the universe radius: $T(r, t) = T(r)e^{-\Lambda t}$, where Λ is the measure of expansion rate.

From the time-dependent equation for local temperature[11]:

$$\frac{\partial \Delta T}{\partial t} - \frac{k}{\rho c_p} \nabla^2 \Delta T = 0 \quad (44)$$

where c_p is specific heat with fixed volume which is negative for black hole, k is heat conductance. As $\Delta T \sim \frac{1}{R(t)} \sim e^{-\Lambda t}$ and $k_B \Delta T \sim \Phi$, we have equation for gravitation potential

energy as $r > r_h$:

$$\begin{aligned}\nabla^2\Phi(r) + \frac{\rho c_p \Lambda}{k}\Phi &= 0, \\ \nabla^2\Phi(r) - \frac{1}{\lambda^2}\Phi &= 0, \\ \lambda = \lambda_{cosmon} &= \left(-\frac{\rho c_p \Lambda}{k}\right)^{-\frac{1}{2}}\end{aligned}\tag{45}$$

$\lambda_{cosmon} \approx 10^{28}cm$ is Compton wave length of cosmic expansion cosmon, its energy and mass are:

$$e_{cosmon} = \frac{2\pi\hbar c}{\lambda} \approx 10^{-45}erg, \quad m_{cosmon} = \frac{2\pi\hbar}{\lambda c} = 10^{-66}g, \quad m_{cosmon}/m_p \approx 10^{-61}\tag{46}$$

As universe expanding, the space-time symmetry of the temperature equation is broken, the radiation excitation quantum acquires mass m_{cosmon} .

In viewing the boundary condition $\Phi(r_h) = -\frac{2GMm}{r_h}e^{-r_h/\lambda}$, the solution of the stationary wave equation (45) is $\Phi(r) = -\frac{2GMm}{r}e^{-r/\lambda}$, and the Einstein potential is

$$\phi(r) = \Phi(r)/m = -\frac{2GM}{r}e^{-r/\lambda} = -\frac{2GM}{r} + \frac{2GM}{r}(1 - e^{-r/\lambda}) \approx -\frac{2GM}{r} + \frac{2GM}{\lambda}\tag{47}$$

(For solar mass, $2GM_\odot/\lambda \sim 10^{-2}(cm/s)^2 \sim 10^{-22}c^2$)

If the cosmic expansion cosmon has the cross section $\sigma = (\frac{\hbar}{m_p c})^2$ and its volume is $V_{cosmon} = \lambda_{cosmon}\sigma$, the mass density of cosmic expansion cosmon is

$$\rho_{cosmon} = \frac{m_{cosmon}}{\frac{\hbar}{m_{cosmon}c}(\frac{\hbar}{m_p c})^2} = \frac{m_{cosmon}^2}{m_p(\frac{\hbar}{m_p c})^3} = \frac{m_p}{(\frac{\hbar}{m_p c})^3} \times 10^{-122} \sim \rho_v \times 10^{-122} = \rho_\Lambda\tag{48}$$

where $m_{cosmon}/m_p \approx 10^{-61}$, $\rho_p = \rho_v \sim m_p/(\frac{\hbar}{m_p c})^3$ is mass density of Planckon or vacuum, and ρ_Λ is just the mass density of dark energy. Eq.(48) indicates that the mass density of cosmic expansion cosmon is equal to the mass density of dark energy. The above result is consistent with the universe expansion model with Panck era as initial condition[12].

V. MICROSCOPIC QUANTUM STATISTICAL STRUCTURE OF GRAVITY

For general gravitation fields, the Einstein potential $\phi(\vec{r})$ is a functional of metrices $g_{\mu\nu}(r)$. If the functional is known,

$$\phi(\vec{r}) = \phi[g_{\mu\nu}(\vec{r})]\tag{49}$$

the gravitation acceleration can be calculated from the Einstein potential:

$$\vec{\kappa}(\vec{r}) = -\nabla\phi(\vec{r}) \quad (50)$$

On the other hand, for a system with spherical symmetry or for a circular orbit, one can calculate the acceleration from zero geodesic trajectory of the radiation particle,

$$\kappa(r) = \sqrt{g_{rr}g_{00}}\frac{d^2r}{d\tau^2} = c^2\sqrt{\frac{g^{rr}}{g_{00}}}\nabla_r g_{00} \quad (51)$$

From mechanical balance $\kappa = \frac{c^2}{r_\kappa}$, we obtain the gravitation curvature radius $r_\kappa = \frac{c^2}{\kappa}$ and the mean energy $e(r_\kappa) = \frac{\hbar c}{2r_\kappa} = \frac{\hbar\kappa}{2c}$ of the corresponding radiation quantum hole which is a standing wave on the r_κ -sphere with wave length $\lambda_\kappa = 4\pi r_\kappa$ and wave vector $k_\kappa = \frac{1}{2r_\kappa}$. From the temperature Green's function[7, 8], $\beta(r_\kappa)e(r_\kappa) = e(r_\kappa)/k_B T(r_\kappa) = \pi$, one obtains the temperature of the gravitation system

$$k_B T(\vec{r}_\kappa) = e(r_\kappa)/\pi = \frac{\hbar c}{2\pi r_\kappa} = \frac{\hbar\kappa}{2\pi c} = \frac{\hbar}{2\pi c}|\vec{\nabla}\phi[g_{\mu\nu}(\vec{r})]| = \frac{\hbar c}{2\pi}\sqrt{\frac{g^{rr}}{g_{00}}}\nabla_r g_{00} \quad (52)$$

Till now, the key information of the microscopic quantum statistical structure of the gravitation system is known: the microscopic constituents of the system are radiation quantum holes with spin 1/2 and mean quantum energy $e(r_\kappa) = \frac{\hbar c}{2r_\kappa}$, the local statistical temperature is $T(r_\kappa)$. In general $T(r_\kappa)$ is local and of space- time dependence, which leads to non-equilibrium, non-balance, and energy flows as discussed in sect.III. Since the Planckon vacuum structure is known, the number of constituents depends on the geometric configuration of the system as shown in black hole cases and can be calculated in the way as the black hole.

VI. SUMMARY AND DISCUSSION

In this article, we have proposed the Planckon densely piled model of vacuum. Based on this vacuum model, the microscopic quantum structure of the Schwarzschild black hole and the quantum statistical origin of its gravity have been studied. It is shown that thermodynamic temperature equilibrium and mechanical acceleration balance make the space-time of the black hole horizon become a singular surface, only radiation particles can exist stably and be confined tightly in the horizon sphere, moving in the horizon with speed of light. For this singular surface, the Casimir effect works inside the horizon. It makes the inside

vacuum have less zero energy than outside vacuum, the temperature difference(outside T_{out} high, inside T_{in} low) and the gravity as its thermal pressure is created. Inside the black hole, each Planckon of the vacuum loses zero energy $e_h = \hbar c/2r_h$, the vacuum energy loss manifests itself as the gravitation strength $\kappa = \frac{2GM}{r_h^2} = \frac{c^2}{r_h}$ and Einstein potential $\phi(R) = -\kappa R$. Then a dual relation between inside and outside regions of the black hole is found. By the dual transformation $R \leftrightarrow \frac{r_h^2}{r}$, the interior potential $\phi(R) = -\kappa R$ is transformed into the exterior potential $\phi(r) = -\frac{2GM}{r} (r > r_h)$. Outside horizon, there exist thermodynamic non-equilibrium and mechanical non-balance which lead to outward centrifugal energy flow and inward gravitation energy flow, their compensation establishes local equilibrium. The lost vacuum energy in the negative gravitation potential regions has been removed to the black hole surface to form a spherical Planckon shell with the thickness $d = 2r_p$. All the particles absorbed by black hole have fallen down to the horizon and converted into radiation quanta consisting of standing waves on the r_h -sphere with spin 1/2 and mean zero energy $e_{h0} = \hbar c/4r_h$, thermodynamic equilibrium and mechanical balance keep them stable and be tightly bound in the horizon. The total mass of the radiation quanta in the horizon becomes the gravitation mass $2M$, half of which is used to compensate the negative gravitation potential energy mass, the physical mass of the black is thus M . The entropy of black hole has been calculated from the microscopic state number of the many-body system consisting of ideal radiation fermion gas and the result is well consistent with Hawking. The four thermodynamic laws of black holes can be expressed in terms of microscopic statistical physics and with the radical modification that the gravitation temperature is constant in the whole inside region not just at the black hole horizon.

The accelerating expansion of the universe yields the expansion cosmon and its energy density agrees with dark energy density.

The results for the black hole have been generalized to general gravitation cases, and the procedures how to find the microscopic quantum statistical structure of a gravitation system are presented.

The study and results of this article are quite preliminary. The Planckon piled vacuum model is constructed quantum-mechanically at mean field and semi-classical level, since the basic brick of vacuum-the Planckon is introduced on average as an ideal quantum object(a standing quantum wave in the Planckon r_p -sphere) and fluctuations around mean values are neglected. The quantization of the black hole excitations is also at semi-classical level,

namely the quantization of vacuum excitations is based on orbital standing waves, just like that N.Bohr had quantized electron motion in hydrogen atom as an orbital standing wave. However, in our case, the spherical black hole plays the role of hydrogen atom. Furthermore, as a quantum statistical system of gravity, the radiation excitation quanta of vacuum are treated as an ideal fermion gas at mean field level, and the interactions among them are not included. In a mean field treatment of a quantum statistical many-body system, quantum and statistical fluctuation around the mean value is not considered. There are two kinds of fluctuations around mean values: 1) quantum mechanical fluctuation around the quantum mechanical mean values and 2) statistical ensemble fluctuations around ensemble average. As the statistical mean energy of radiation quantum, it is naturally related to a temperature by the period condition of temperature Green's function: $e_h/\pi = k_B T_h$. As a result, one should consider the Dirac Probability distribution $f(\frac{e-\mu}{k_B T_h}) = \frac{1}{e^{(e-\mu)/k_B T_h} + 1}$ with $T_h = \frac{\hbar c}{2\pi k_B r_h}$ for radiation fermion quanta. For the quantum mechanical fluctuation, one should consider the quantum zero energy fluctuation and the effective temperature due to zero energy fluctuation should be introduced with $k_B T_p \sim e_p/\pi$ for Planckon energy distribution. Since the radiation quanta correspond to extremely long classical trajectories and low energies, and the number of radiation quanta is extremely large, both fluctuations are negligibly small. That is why we obtain the exact quantities for the black hole.

In our model, vacuum is a kind of Planckon densely piled liquid crystal, its radiation excitations are like phonons and its particle excitations are like dislocations or defects in solid[6]. Thus the investigation of the relation between gravity and solid states is very interesting and significant[13]. In this respect, it has been shown that vacuum is a kind of super fluid medium[14]. It should be noted that our model may be related to super-string theory[15] and loop quantum gravity theory[9] in some respects. The detailed relation between our microscopic model and the macroscopic theory of general relativity is quite worthy to explore and the generalization of this study to different kinds of black holes is quite desirable.

Appendix A: Dual relation between inside and outside gravity of Schwarzschild black hole

Dual relation of coordinates:

$$R \leftrightarrow r, \quad R = \frac{r_h^2}{r} \quad (\text{A1})$$

Dual relation of inside and outside regions :

$$r \in [r_h, \infty) \leftrightarrow R \in [0, r_h] \quad (\text{A2})$$

Dual relation Einstein potential:

$$\phi(r) = -\frac{2GM}{r} = -\frac{c^2 R}{r_h} = \phi(R) \quad (\text{A3})$$

Dual relation of gravity strength: outside $\kappa(r)$ pointing to horizon

$$\kappa = -\frac{d\phi(r)}{dr} = -\frac{2GM}{r^2} = -\frac{c^2}{r_\kappa}, \quad r_\kappa = r\left(\frac{r}{r_h}\right) \quad (\text{A4})$$

inside $\kappa(R)$ pointing to horizon

$$\kappa(R) = -\frac{d\phi(R)}{dR} = \frac{c^2}{r_h} \quad (\text{A5})$$

Horizon surface behaves as an attractor surface!

The problem of the transport of exterior vacuum energy loss due to existence of negative potential energy to surface can be converted into interior problem. By virtue of the dual transformation, the exterior gravity potential and its strength can be transformed into internal ones. From the formulae $\pi k_B \Delta T = \frac{\hbar \kappa(R)}{2c} = \frac{\hbar c}{2r_h} = e_h$ ($\kappa(R) = \frac{c^2}{r_h}$), we know that every Planckon inside vacuum loses energy e_h . Total number of Planckons in this region is $N_T = (\frac{r_h}{r_p})^3$, so total energy removed from this region to surface is : $E_T = e_h (\frac{r_h}{r_p})^3 = \frac{1}{2} \frac{\hbar c}{4r_p} 4(\frac{r_h}{r_p})^2 = \frac{1}{2} e_{p0} N_h$, where $N_h = 4(\frac{r_h}{r_p})^2$ is the number of Planckons in the shell of black surface and $e_{p0} = \frac{\hbar c}{4r_p}$ is Planckon zero energy. The result is the same as that obtained from direct calculation in section III.

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